Supplementary Document:
Robust Subspace Discovery through Supervised Low-Rank Constraints

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This document includes the following supplementary materials to the main text of the paper.

- Section 1: proof of Theorem 3.1.
- Section 2: experimental data descriptions.

1 Proof of Theorem 3.1

Theorem 3.1. If $\eta > 0$, the supervised regularization term $f(P,Z) = \|P^T AZ(I - I_b)\|_F^2 - \|P^T AZ(I_b - I)\|_F^2 + \eta \|P^T AZ\|_F^2$ is strictly convex to $Z$ when $P$ is fixed.

Proof. Let $T = P^T AZ$, where $P^T A$ can be regarded as constant when optimizing $Z$. We then can convert $f(P,Z)$ to $f(T)$ as follows

\[ f(T) = \|T(I - I_b)\|_F^2 - \|T(I_b - I)\|_F^2 + \eta \|T\|_F^2. \]

Now we can rewrite $T$ as a column vector, $T = [r_1, r_2, \ldots, r_n]^T$, where $r_i$ is the $i$-th row vector of $T$. Then $f(T)$ is equivalent to

\[ f(T) = \|\text{diag}((I - I_b)^T)T\|_2^2 - \|\text{diag}((I_b - I_t)^T)T\|_2^2 + \eta \|T\|_2^2, \]

where $\text{diag}(K)$ is to construct a block diagonal matrix with each block on the diagonal being matrix $K$.

The convexity of $f(T)$ depends on whether its Hessian matrix $\nabla^2 f(T)$ is positive definite or not. $\nabla^2 f(T)$ will be positive definite if matrix $S$ is positive definite.

\[ S = (I - I_b)(I - I_b)^T - (I_b - I)(I_b - I)^T + \eta I. \]

After some derivations, we have

\[ S = (1 + \eta)I - 2E_1/n_1 + E_2/n, \]

where $E_1$ and $E_2$ are all the matrices of size $n \times n$. $E_1(i,j)$ = 1 only if the $i$-th sample and $j$-th sample belong to the same class, and all the entries in $E_2$ are equal to 1.

In order to make $S$ positive definite, all the eigenvalues of $S$ should be greater than 0. Since the maximal eigenvalues of $E_1$ and $E_2$ are $n_1$ and $n$, we should ensure:

\[ (1 + \eta) - 2n_1/n_1 + n/n > 0. \]

Hence, we have $\eta > 0$ from the above equation, which could guarantee that $f(T)$ is convex to $T$. Recall that $T = P^T AZ$ and $P^T A$ is a constant. Therefore, we can further conclude that $f(P,Z)$ is convex to $Z$ when $\eta > 0$ and $P$ is fixed.

2 Data Descriptions

In Section 5 of the main text of this paper, we present comprehensive evaluations on the following four benchmark data sets. The following details the collection and preprocessing of the four datasets.

- **COIL-100 Object Dataset**: The COIL dataset contains various views of 100 different objects under controlled views and lighting conditions. For each object in the database, there are 72 equally spaced views, which are obtained as the camera circles the object. In our experiments, the images are converted to grayscale, resized to $32 \times 32$, and then the robustness is evaluated on alternative viewpoints. In addition, to test the scalability of our method, we increase the number of classes from 20 to 80, and compare the classification performance of different methods. Unlike most existing subspace learning experiments, we also test the robustness of different methods to noise by adding 10% pixel corruption to the original images. Figure 1(a) shows some examples of original and corresponding corrupted images in COIL dataset.

- **ALOI Object Dataset**: The ALOI dataset contains 1000 general object categories taken at different viewing angles. There are 72 equally spaced views in each category. In our experiments, we select the first 300 objects from this dataset. All the images are converted to grayscale and resized to the size of $36 \times 48$. We also add 10% pixel corruption on the original images to evaluate the performance of different methods.

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1 http://www.cs.columbia.edu/CAVE/software/softlib/coil-100.php

2 http://staff.science.uva.nl/ aloi/
Figure 1: Example images of two object datasets: (a) COIL dataset, (b) ALOI dataset.

Figure 2: Sample images in YaleB, FERET and KinFace datasets.

Figure 1(b) shows some examples of the original and corrupted images in the ALOI dataset.

**Extended YaleB Face Dataset**\(^{3}\): The Extended YaleB face dataset \([1]\) consists of 2414 frontal face images of 38 classes, and each of them contains about 64 images. Figure 2(a) shows the examples from the YaleB dataset. We crop and resize the images to the size of \(28 \times 32\), and normalize the pixel values to \([0, 1]\).

**FERET Face Dataset**\(^{4}\): The FERET database \([2]\) contains 2,200 facial images from 200 individuals with each person contributing 11 images. The images, named with two-character strings ranging from "ba" to "bk", were captured under various illuminations and display a variety of facial expressions and poses. In this experiment, we randomly select the images from 50 individuals. Figure 2(b) provides images of one individual that show large pose variations. Each image is \(384 \times 256\) with 256 gray levels. Each image was cropped and resized to \(30 \times 25\).

**References**


\(^{3}\)http://vision.ucsd.edu/~leekc/ExtYaleDatabase/ExtYaleB.html

\(^{4}\)http://www.face-rec.org/databases